Fluid flow

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Fluids

- Fluids – continually deform (flow) under shear stress
  - Liquid
  - Gas
  - Plasma

- Liquids
  - Short-range crystalline organisation that always reforms
  - Constant volume, incompressible
  - Moderate resistance to deformation: fit the shape of solid surrounding (container) or force field
  - No preferred direction

Fluid mechanics

- Hydrostatics (resting fluids)
- Hydrodynamics (moving fluids)
  - Ideal fluids (no internal friction)
  - Real (viscous) fluids
    - Newtonian fluids
    - Non-newtonian fluids
- Flow
  - Laminar flow
  - Turbulent flow
    - Stationary flow (equal amounts of mass or volume through the cross section in a unit of time)
    - Changing in time

Pascal’s law

\[ p = \frac{F}{A} \]

\[ W = p \cdot \Delta V \]

Fluids are incompressible:

\[ A_1 \cdot d_1 = \Delta V = A_2 \cdot d_2 \]

\[ p_1 \cdot A_1 \cdot d_1 = W_1 = W_2 = p_2 \cdot A_2 \cdot d_2 \]

\[ p_1 = p_2 \]

\[ \frac{F_1}{A_1} = \frac{F_2}{A_2} \]

\[ A_1 < A_2 \rightarrow F_1 < F_2 \]

Pressure exerted anywhere on a fluid in a confined space is transmitted to an equal extent in all directions.

Blaise Pascal (1623-1662, FRA)
Hydrostatic pressure

- In a gravitation field: pressure is proportional to height (depth) because of the weight of the fluid column.

\[ F = G = m \cdot g \]

\[ p = \frac{F}{A} = \frac{m \cdot g}{A} = \rho \cdot V \cdot g = \rho \cdot h \cdot A \cdot g = \rho \cdot h \cdot g \]

1 mmHg (1 Torr) = 133.3 Pa
1 atm = 101 325 Pa = 101 kPa

- If also atmospheric pressure is considered:

\[ p = p_{atm} + \rho \cdot h \cdot g \]

- For interconnected fluids in equilibrium:

\[ \frac{h_1}{h_2} = \frac{\rho_1}{\rho_2} \]

- Independent of the fluid’s shape

Law of Archimedes

- Objects immersed in fluid lose weight

\[ F_1 = p_1 \cdot A = \rho \cdot g \cdot \rho_{fluid} \cdot h_1 \cdot A \]

\[ F_2 = p_2 \cdot A = \rho \cdot g \cdot \rho_{fluid} \cdot h_2 \cdot A \]

\[ F_{ext} = F_2 - F_1 = \rho \cdot g \cdot \rho_{fluid} \cdot (h_2 - h_1) \cdot A = \rho \cdot g \cdot \rho_{fluid} \cdot V = \rho \cdot m_{fluid} \]

Flow

- Movement of fluids in one direction

- Driving force is the pressure difference

- Foundational axioms: conservation laws (mass, energy, momentum)

- Continuum assumption:
  - Fluids are continuous matter rather than made up of molecules.
  - Physical properties are well-defined at infinitesimal points, and vary continuously from one point to another.

- intensity of current or volumetric flow rate

\[ I_v = \frac{V}{t} = \frac{\Delta V}{\Delta t} = \frac{m^3}{s} \]

- 5L/min in aorta
Law of continuity

- Fluids are incompressible
  
  In \( \Delta t \) time the flow volume through any cross-section is the same:
  
  \[
  I = \frac{\Delta V}{\Delta t} = \frac{A \cdot \Delta v}{\Delta t} = A \cdot v
  \]

  \( I_1 = I_2 \quad A_1 \cdot v_1 = A_2 \cdot v_2 \)  \text{ continuity equation}

- For rigid tubes, ideal fluids and stationary flow: (conservation of mass)
  
  \( I = A \cdot v = \text{constant} \)

Bernoulli’s law

- mechanical work: \( W = F \cdot \Delta x \)
  
  \( A_1 \cdot x_2 = A_2 \cdot x_2 = \Delta V \)
  
  \( W = F \cdot \frac{\Delta V}{A} = F \cdot \frac{\Delta V}{A} = \rho \cdot \Delta V \)

- conservation of energy:
  
  \( \Delta W = \Delta E_{\text{kin}} \)
  
  \[
  W_1 - W_2 = E_{k1} - E_{k2}
  \]
  
  \[
  W_1 + E_{k1} = W_2 + E_{k2}
  \]
  
  \[
  p_1 \cdot \Delta V + m \cdot \frac{v_1^2}{2} = p_2 \cdot \Delta V + m \cdot \frac{v_2^2}{2}
  \]
  
  \[
  \frac{\Delta V}{\Delta V} \Rightarrow p_1 + \rho \cdot \frac{v_1^2}{2} = p_2 + \rho \cdot \frac{v_2^2}{2}
  \]

Bernoulli’s law

- potential energy:
  
  \[
  \Delta(m \cdot g \cdot h) = m \cdot g \cdot h_2 - m \cdot g \cdot h_1
  \]
  
  \[
  p_1 \cdot \Delta V + m \cdot \frac{v_1^2}{2} + m \cdot g \cdot h_1 = p_2 \cdot \Delta V + m \cdot \frac{v_2^2}{2} + m \cdot g \cdot h_2
  \]
  
  \[
  \frac{\Delta V}{\Delta V} \Rightarrow p_1 + \rho \cdot \frac{v_1^2}{2} + \rho \cdot g \cdot h_1 = p_2 + \rho \cdot \frac{v_2^2}{2} + \rho \cdot g \cdot h_2
  \]

- hydrostatic pressure

  \[
  p + \rho \cdot \frac{v^2}{2} + \rho \cdot g \cdot h = \text{constant}
  \]

Laminar flow in real fluids

- constant cross-section
- pressure decreasing in the direction of flow
- proportional to distance

- Fluid resists to the moving effect (flow) with a force
- An internal friction between imaginary fluid layers sliding past each other \( \Rightarrow \) decreasing displacement profile

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Newton’s law

\[ F = \Delta F \cdot \frac{\Delta x}{\Delta y} \]

\[ \text{shear stress} = \frac{F}{A} \]

\[ \text{strain rate} = \frac{\Delta v}{\Delta y} = \frac{\Delta x}{\Delta y} \]

\[ \text{viscosity} = \frac{\text{shear stress}}{\text{strain rate}} = \frac{F}{A} \cdot \frac{\Delta y}{\Delta x} \]

\[ \frac{\eta}{m^2 \cdot s} = \frac{N \cdot m}{m^2 \cdot s} = \frac{p \cdot s}{m^2} \]

Isaac Newton (1643-1727, ENG)

Viscosity depends on matter, temperature, concentration, pressure

- Ideal fluid: zero viscosity
  (~ certain liquid He species)

- Newtonian-fluid (e.g., water):
  - Viscosity ~ shear stress

- Non-newtonian fluid (e.g., blood):
  - Viscosity not proportional to shear stress
  - It depends on flow velocity

- Viscosity of gases increases at higher temperatures, that of fluids decreases

Hagen-Poiseuille law

\[ \Delta p \cdot A = (p_1 - p_2) \cdot A = F = -\eta \cdot A \cdot \frac{\Delta v}{\Delta h} \]

\[ I = \frac{\eta \cdot I}{\pi \cdot r^4} \]

\[ \frac{1}{r^4} \frac{\Delta p}{L} \]

\[ I = \frac{Q}{8 \cdot \eta} \]

\[ R = \frac{8 \cdot \eta \cdot I}{\pi \cdot r^4} \]

\[ \Delta p = I \cdot R \]

Goethe Hagen (1797-1884, GER)

Jean Poiseuille (1797-1869, FRA)

Turbulent flow

- Laminar flow
  - Low speed
  - No swirling
  - On smooth surface

- Turbulent flow
  - High speed for viscosity
  - Swirling, no “layers”
  - On rough surface (blood vessels)

- Reynolds number

\[ R = \frac{\eta \cdot q \cdot \rho}{\eta} \]

\[ \frac{v_{crit}}{q} \]

\[ \text{critical velocity for smooth tubes} \]

\[ R_{crit} = 1160 \]

Osborne Reynolds (1842-1912, IRE)
Hydrostatic resistance

- A medium (gas or liquid) exerts an opposite force to the direction of the movement on the objects moving in it:

\[ F = k \cdot A \cdot \rho_{\text{medium}} \cdot \nu^2 \]

- Streamline bodies (small k):
  - flow layers unite behind the object, low resistance

- Non-streamline bodies (great k):
  - the medium flows rapidly behind the object
  - low pressure → suction effect → great counter-force

Stokes’ law

- Movement of bodies in real fluids

Frictional force on moving sphaerical objects:

\[ F = 6 \cdot \pi \cdot \eta \cdot r \cdot \nu \]

- Terminal (settling) velocity:

\[ \nu_{\text{crit}} = \frac{12 \cdot \eta}{g \cdot r} \]

\[ \nu = \frac{2}{9} \left( \frac{\rho_{\text{obj}} - \rho_{\text{fluid}}}{\eta} \right) \cdot g \cdot r^2 \]

Summary

- Pascal’s law: transmittion of pressure
- Continuity equation: relationship of surface and flow velocity

\[ I = A \cdot \nu = \text{constant} \]

- Bernoulli’s law: relationship of pressure and velocity

\[ p + \rho \cdot \frac{\nu^2}{2} + \rho \cdot g \cdot h = \text{constant} \]

- Newton’s law: internal friction

\[ F = \eta \cdot A \cdot \frac{\Delta \nu}{\Delta x} \]

- Hagen-Poiseuille law: flow of real fluids

\[ I = (Q / \pi \cdot r^4) \cdot \frac{\Delta p}{\Delta x} \]

- Reynolds-number: critical velocity of the turbulent flow

\[ R = \nu \cdot \frac{\rho \cdot r^2}{\eta} \]

- Stokes’ law: objects moving in a medium

\[ F = 6 \cdot \pi \cdot \eta \cdot \nu \cdot r \]

THANK YOU FOR ATTENTION!
Torricelli's law

Specific case of Bernoulli's law

at the top and at the opening: \( p = p_{\text{atm}} \)

at the top: \( v = 0 \)  at the opening: \( h = 0 \)

\[ g \cdot h \cdot p + p_{\text{atm}} = \rho \cdot \frac{v^2}{2} + p_{\text{atm}} \]

\[ v = \sqrt{2 \cdot g \cdot h} \]

Evangelista Torricelli (1608-1647, ITA)

- Venturi effect
  - Flow through a constriction
  - Gain in kinetic energy
  - Loss in pressure

  perfume spray, chimney

Giovanni Venturi (1746-1822, ITA)