DIFFUSION
Transport processes: processes in which the spatial distribution of a physical quantity is altered or translocated

- ENERGY: HEAT (Q) / WORK (W)
- MATTER: PARTICLES (m)
GENERAL DESCRIPTION

intensive physical quantity: \( x \) (temperature (T), pressure (p), chemical potential (\( \mu \)), electrostatic potential (\( \phi \)), …)

extensive physical quantity: \( y \) (entropy (S), volume (V), mass (m), electric charge (Q), …)

inhomogeneity in an intensive physical quantity:

- gradient of \( x \)

\[
X = - \frac{\Delta x_{\text{intensive}}}{\Delta r}
\]

provides force that drives the flow of the corresponding extensive quantity \( y \)

homogeneity in the intensive physical quantity

\( x = \text{constant} \)

\[
X = 0
\]
GENERAL DESCRIPTION

$\Delta y$ extensive physical quantity flows through a surface of $A$ (perpendicular to the direction of the transport) during $\Delta t$

\[ I_v = \frac{\Delta y}{\Delta t} \]

flow rate

\[ J = \frac{I_v}{A} = \frac{\Delta y}{A\Delta t} \]

flow density

ONSAGER’S EQUATION: linear, irreversible processes

$Lars$ $Onsager$ $(1903-1976$, NOR-USA, Nobel-prize, 1968)$

$J$: flow density of the extensive physical quantity ($y$)

$X$: thermodynamic force gradient of the intensive physical quantity ($x$)

$L$: conductivity coefficient

flow density of the extensive quantity ($J$) is linearly proportional to the gradient of the intensive quantity ($X$)
MOLECULAR MOTION

Brownian motion
- Robert Brown (1773-1858, SCO)
- agitation of pollen in a drop of water
- random (zigzag) thermal motion of particles in suspension
- results from the continuous collisions between the particles and the molecules of the suspension

3D Brownian motion of a particle
DIFFUSION

- determined by the inhomogenous distribution of particles
- gradient of the intensive quantity: concentration (c), chemical potential (µ)
- caused by the random thermal motion of particles
- is a net transport of particles from a region of higher concentration to a region of lower concentration by random molecular motion
- continues until the distribution of particles is uniform

\(\Delta n\) (amount of substance in moles) quantity flows through a surface of \(A\) (perpendicular to the direction of the transport) during \(\Delta \tau\):

\[
I_v = \frac{\Delta n}{\Delta \tau}
\]

**matter flow rate**

depends on \(A\)

unit: mol/s

\[
J = \frac{I_v}{\Delta A} = \frac{\Delta n}{\Delta A \Delta \tau}
\]

**matter flow density**

independent of \(A\)

unit: mol/m\(^2\)/s
What does the „strength” of diffusion depend on?

Let’s quantify diffusion in 1D (along the x axis)

spatial variation of the concentration along the x axis: \( c(x) \)

assumption: linear change in the concentration

\( \Delta c/\Delta x = \text{constant} \)
Onsager’s linear equation

\[ \Delta \tau = \frac{X}{\mu - \Delta \mu} \]

**Onsager’s linear equation**

\[ J \approx X \]

\[ J = \frac{I_v}{\Delta A} = \frac{\Delta n}{\Delta A \Delta \tau} \]

\[ X = \frac{\Delta c}{\Delta x} \]

**FICK’S 1ST LAW**

matter flow density is linearly proportional to the drop in concentration

- negative sign: diffusion current is in the direction where the concentration drops
- \( D \): diffusion coefficient

Adolf Fick (1829-1901, GER)
DIFFUSION COEFFICIENT

\[
D = -\frac{\Delta n}{A\Delta \tau \Delta c} \quad [D] = \frac{mol}{m^2s} = \frac{m^2}{s}
\]

- the amount of substance that diffuses through a surface unit during a time unit if the concentration drop was unity

- the diffusion coefficient D tells us how ‘fast’ a given substance is wandering

- depends on both the diffusing particle and the medium in which the particle diffuses
DIFFUSION COEFFICIENT

For spherical particles \( r \) in a viscous medium \( \eta \) at \( T \) temperature:

\[
D = \frac{kT}{6\pi\eta r}
\]

**STOKES-EINSTEIN EQUATION**

- temperature \( (T) \)
  the higher the temperature, the stronger the thermal motion

- geometry of the particle
  globular proteins diffuse more easily than fibres

- molar mass of the particle \( (M) \)
  heavier particles diffuse more slowly

- viscosity of the medium \( (\eta) \)
  diffusion is faster in gases than in liquids
### DIFFUSION COEFFICIENT

<table>
<thead>
<tr>
<th>diffusing particle [molecular mass (MW)] = gmol(^{-1})/geometry</th>
<th>medium</th>
<th>([D] = m^2s^{-1}) T = 20°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{H}_2 ) (2)</td>
<td>air</td>
<td>(6.4 \cdot 10^{-5})</td>
</tr>
<tr>
<td>( \text{O}_2 ) (32)</td>
<td>air</td>
<td>(2 \cdot 10^{-5}) x3</td>
</tr>
<tr>
<td>( \text{O}_2 ) (32)</td>
<td>water</td>
<td>(1.9 \cdot 10^{-9}) x10000</td>
</tr>
</tbody>
</table>

- **glycine (amino acid)**
  - MW: 75
  - water
  - \(0.9 \cdot 10^{-9}\) x10

- **serum albumin (globular)**
  - MW: 69 000 / 60 x 96 x 60 Å
  - water
  - \(6 \cdot 10^{-11}\) x3

- **tropomyosin (fibrilar)**
  - MW: 65 000 / \(l = 400\) Å
  - water
  - \(2.2 \cdot 10^{-11}\)

- **tobacco mosaic virus**
  - MW: 40 000 000 / \(l = 3000\) Å \(d = 150\) Å
  - water
  - \(4.6 \cdot 10^{-12}\)
TIME AND SPACE-DEPENDENCE OF DIFFUSION - FICK’S 2\textsuperscript{ND} LAW

Let’s assume a tiny volume ($\Delta V$), where the concentration ($c$) is constant in space, so we only have to consider the time dependence: $c(t)$

\[
\Delta V = A\Delta x
\]

particles diffusing IN

\[
J_x = \frac{\Delta n_{IN}}{\Delta A \Delta t}
\]

matter flow density

\[
\Delta n_{IN} \approx J_x A \Delta t
\]

particles diffusing OUT

\[
J_{x+\Delta x} = \frac{\Delta n_{OUT}}{\Delta A \Delta t}
\]

matter flow density

\[
\Delta n_{OUT} \approx J_{x+\Delta x} A \Delta t
\]

1. How does the amount of particles change ($\Delta n$) in the selected volume ($\Delta V$) during $\Delta t$?

The amount of particles ($\Delta n$) diffusing in and out during $\Delta t$:

\[
\Delta n_{IN} \approx J_x A \Delta t
\]

\[
\Delta n_{OUT} \approx J_{x+\Delta x} A \Delta t
\]

The net change in the amount of particles ($\Delta n$) in the selected volume ($\Delta V$) during $\Delta t$:

\[
\Delta n = \Delta n_{IN} - \Delta n_{OUT} \approx J_x A \Delta t - J_{x+\Delta x} A \Delta t = (J_x - J_{x+\Delta x}) A \Delta t
\]
TIME AND SPACE-DEPENDENCE OF DIFFUSION - FICK'S 2\textsuperscript{ND} LAW

2. How does the amount of particles change ($\Delta n$) in the selected volume ($\Delta V$) during $\Delta t$?

From the definition of the concentration:

$$c = \frac{n}{V} \rightarrow \Delta n = \Delta c V$$

The change in the concentration ($\Delta c$) in $\Delta V$ during $\Delta t$:

$$\Delta c = c_{t+\Delta t} - c_t$$

The net change in the amount of particles ($\Delta n$) in the selected volume ($\Delta V$) during $\Delta t$:

$$\Delta n = (c_{t+\Delta t} - c_t) \Delta V = (c_{t+\Delta t} - c_t) A \Delta x$$

$$\Delta V = A \Delta x$$
The net change in the amount of particles ($\Delta n$) in $\Delta V$ during $\Delta t$:

- calculated from the concentration ($c$) = calculated from the matter flow density ($J$)

\[
(c_{t+\Delta t} - c_t) A \Delta x = (J_x - J_{x+\Delta x}) A \Delta t
\]

\[
\Delta c \Delta x = -\Delta J \Delta t
\]

Fick’s 1st law

\[
J = -D \frac{\Delta c}{\Delta x}
\]

\[
\frac{\Delta (D \frac{\Delta c}{\Delta x})}{\Delta x} = \frac{\Delta c}{\Delta t}
\]

Fick’s 2nd law

\[
D \frac{\Delta c}{\Delta x} = \frac{\Delta c}{\Delta t}
\]
How far does a particle get from its initial position during $t$?

$R(t) = ?$

$R(t) \approx \sqrt{t} = \sqrt{3Dt}$

Diffusion length ($R$):
the distance in which the concentration falls to the
$1/e$ of its original value ($c_0$)

- diffusion coefficient
- time

EXAMPLE: 1D FREE DIFFUSION

- solving Fick's 2nd law is difficult
- special case for 1D:

$$c(x, t) = \frac{n_0}{2\sqrt{\pi Dt}} e^{-\frac{x^2}{4Dt}}$$
OSMOSIS
OSMOSIS

1. WALL

NO TRANSPORT

the distribution of particles does not change
OSMOSIS

2. NO WALL

free DIFFUSION
both particles (smaller/larger) reach equal distributions
3. SPECIAL WALL

**semipermeable**
allows smaller solvent molecules to pass through, but not the larger solute molecules → "filter"
pl: animal skin pellicles, walls of living cells, ceramic plate with holes, cellophane

**restricted DIFFUSION – OSMOSIS**
smaller molecules reach a uniform distribution
larger molecules remain in the compartment
OSMOTIC PRESSURE

low solute  high solute

semipermeable membrane

concentration difference + semipermeable membrane
concentration difference + semipermeable membrane

→ solvent (water) flows through the semipermeable membrane

\[ J_{\text{IN}} > J_{\text{OUT}} \]
OSMOTIC PRESSURE

low solute  high solute

concentration difference + semipermeable membrane

→ solvent (water) flows through the semipermeable membrane  \( J_{IN} > J_{OUT} \)

→ the volume of the more concentrated solution increases (height of the liquid column: \( h \))
low solute  high solute

**OSMOTIC PRESSURE**

- Concentration difference + semipermeable membrane
- Solvent (water) flows through the semipermeable membrane: $J_{IN} > J_{OUT}$
- The volume of the more concentrated solution increases (height of the liquid column: $h$)
- The pressure increases in the more concentrated solution: hydrostatic pressure $\sim \rho$, $h$

$\rightarrow$ *dynamic equilibrium*: for solvent flow $J_{IN} = J_{OUT}$  

**OSMOTIC EQUILIBRIUM**
concentration difference + semipermeable membrane
→ solvent (water) flows through the semipermeable membrane  \( J_{IN} > J_{OUT} \)
→ the volume of the more concentrated solution increases (height of the liquid column: \( h \))
→ the pressure increases in the more concentrated solution: hydrostatic pressure

→ dynamic equilibrium: for solvent flow  \( J_{IN} = J_{OUT} \)  **OSMOTIC EQUILIBRIUM**

**OSMOTIC PRESSURE**  
\[ p_{osmotic} = \rho gh \]

pressure that has to be exerted on the solution connected to pure solvent by a semipermeable membrane to reach dynamic equilibrium, to counteract osmosis
Osmotic Pressure

For dilute solutions and perfect semipermeable membranes, the equation of state of the ideal gas applies:

\[ p_{\text{osmotic}} V = nRT \]

\[ p_{\text{osmotic}} = \frac{n}{V} RT \]

Osmotic pressure is given by:

\[ p_{\text{osmotic}} = cRT \]  \textbf{Van’t Hoff Law}

The osmotic pressure exerted by any substance in dilute solution is the same as that it would exert if present as gas in the same volume:

\[ p_{\text{osmotic}} \approx c \]

Jacobus van’t Hoff (1852-1911, NED)
different (macro)molecules are sorted by semipermeable membranes
pore size of the membrane → the limit in molecular mass of which molecules can pass through the membrane
CLASSIFYING SOLUTIONS ON THE BASIS OF OSMOTIC PRESSURE

same osmotic pressure: **ISOTONIC**
extra- and intracellular solutions with the same osmotic pressure
the osmotic pressure of the solutions in the cells of human body = osmotic pressure of a 0.87 % (n/n) (0.15 M) NaCl solution → physiologic saline solution

higher osmotic pressure: **HYPERTONIC**
extracellular solution has higher osmotic pressure than the intracellular solution → water efflux

lower osmotic pressure: **HYPOTONIC**
extracellular solution has lower osmotic pressure than the intracellular solution → water influx
mammalian cells: \( p_{\text{osmotic}} = 0.8 \cdot 10^6 \text{ Pa} \)

\( p_{\text{atm}} = 10^5 \text{ Pa} \)

**HYPERTONIC**
(more concentrated: 10% NaCl)

**IZOTONIC**
(0.9 % NaCl)

**HYPOTONIC**
(less concentrated: 0.01% NaCl)

passive water efflux

passive water influx
HYPERTONIC
HYPOTONIC
IZOTONIC (0.9 % NaCl)
RED BLOOD CELLS IN DIFFERENT ENVIRONMENT

BIOPHYSICS I - OSMOSIS
plant cells: \( p_{\text{osmotic}} = 0.4 - 4 \cdot 10^6 \, Pa \)

**HYPERTONIC**
- **PLAZMOLYSIS**
  - plasma membrane pulls away from the cell wall

**IZOTONIC**
- **TURGOR PRESSURE**
  - plasma membrane pushed to the cell wall (turgor pressure)

**HYPOTONIC**
IN THE MEDICAL PRACTICE

**injection, infusion**
physiologic saline solution

**oedemas, inflamed areas**
dextran-solution / bitter salt (MgSO$_4$-solution): hypertonic compared to the fluids of the body $\rightarrow$ water efflux

**laxative salts**
barely absorbed by the large intestine, thus they create hypertonic conditions which causes water influx into the large intestine $\rightarrow$ dilution of colonic content, facilitate excretion
IN THE MEDICAL PRACTICE

**haemodialysis**
remove soluble chemicals toxic for the body
BIOLOGICAL RELEVANCE
TRANSPORT ACROSS CELL MEMBRANES

**transport mechanism**

*without transporter molecule*
- diffusion

*with transporter molecule*
- facilitated diffusion
- carrier-protein

**energetic requirements**

*passive transport*
- oxygen

*active transport*
- sodium
- glucose
- ATP
- ADP
- inorganic phosphate
Passive transport: DIFFUSION
the particle transport is determined by the concentration gradient

rate is determined by
- concentration gradient
- temperature
- the size and shape of the diffusing particle
- size of the surface
- distance

hydrophobic molecules: O2, N2
small polar molecules: CO2, water, alcohol, urea, glycerine
glucose, saccharose
Passive transport: DIFFUSION OF IONS ACROSS ION CHANNELS

the particle transport is determined by the concentration- and electric potential gradient

*electrochemical potential gradient*

*ion-channels*: transmembrane proteins (pore)
closed state: no transport
open state: transport

*selectivity*: charge and size of the ion
BIOLOGICAL RELEVANCE
TRANSPORT ACROSS CELL MEMBRANES

Passive transport: FACILITATED DIFFUSION
the particle transport is determined by the concentration gradient

carrier-proteins (mediator, transporter)
specifically bind the ions or molecules and promote their transport
transport: reversible conformational change in the carrier-protein